5/H–28 (v) (Syllabus–2015)

2 0 1 8

(October)

STATISTICS

(Honours)

(Mathematical Methods and Distribution Theory)

[STH–51 (TH)]

Marks : 56

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* What do you mean by numerical integration? What is the approach of deriving numerical integration? Explain briefly with an example. 1+3=4

   (b) State and obtain Weddle's rule of numerical integration by deriving general quadrature formula. 8

2. (a) Give the analytical definition of maxima and minima of a function.

(b) Write the conditions under which $f(x)$ is a maximum or a minimum at $x = a$.

(c) Find for what values of x, the expression $x^3 - 9x^2 + 24x - 12$ is a maximum or a minimum.

UNIT—II

3. (a) Define inverse of a square matrix. If A and B are non-singular square matrices of the same order, then show that

$$(AB)^{-1} = B^{-1}A^{-1}$$

(b) Write a note on 'solution of linear system of equations'.

(c) Show that a system of m linear equations in n unknowns will be consistent if and only if the coefficient matrix and the augmented matrix have the same rank.

4. Define the following : 2+2+1+2+4=

(a) Vector spaces and subspaces

(b) Sum of two n-vectors and multiplication of a vector by a scalar

(c) Linear combination of vectors

(d) Linear dependence and independence of vectors with examples

(e) Quadratic forms and properties of a quadratic form

UNIT—III

. (a) Define two-dimensional random variables, two-dimensional distribution function and marginal distribution function. What do you mean by 'independent random variables'? 4

(b) Joint distribution of X and Y is given by

$$f(x, y) = 4xy\, e^{-(x^2 + y^2)}\ ;\ x \geq 0,\ y \geq 0$$

Test whether X and Y are independent. For the above joint distribution, find the conditional density of X given $Y = y$. 7

5. (a) Define expected value of a random variable. Write its properties. 3

(b) What is covariance? For the n random variables $X_1, X_2, \cdots, X_n$, show that

$$V\left(\sum_{i=1}^{n} a_i X_i\right) = \sum_{i=1}^{n} a_i^2 V(X_i) + 2 \sum_{i=1}^{n} \sum_{\substack{j=1 \\ i<j}}^{n} a_i a_j \operatorname{cov}(X_i X_j)$$

1+3=4

(c) (i) Find the expectation of the number on a die when thrown.

(ii) Two unbiased dice are thrown. Find the expected value of the sum of numbers of points on them.

UNIT—IV

7. Define gamma distribution. Write its cumulative distribution function.

Obtain the moment-generating function of gamma distribution and hence show that the sum of independent gamma variate is also a gamma variate.

Derive the cumulant-generating function of gamma distribution and obtain its mean, variance, β_1 and β_2.

2+4+5=11

8. Define beta distribution of first and second kinds. What do you mean by incomplete beta function?

Obtain the mean, variance, β_1 and β_2 of the beta distribution of first kind.

Derive the harmonic mean of the beta distribution of first kind and second kind.

3+4+4=11

(Continued)

UNIT—V

9. Define chi-square variate and derive its distribution.

Obtain moment-generating function of chi-square distribution.

Find mean, variance, β_1 and β_2 from cumulant-generating function of chi-square distribution.

6+2+3=11

10. (a) Define Student's t-statistic and derive its probability density function. 5

(b) Define F-statistic and mention its properties. Establish the relationship between F and chi-square distributions. 6

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